Interdomain Routing and Games

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ABSTRACT

We present a game-theoretic model that captures many of the intricacies of *interdomain routing* in today's Internet. In this model, the strategic agents are source nodes located on a network, who aim to send traffic to a unique destination node. The interaction between the agents is dynamic and complex – asynchronous, sequential, and based on partial information. Best-reply dynamics in this model capture crucial aspects of the interdomain routing protocol de facto, namely the Border Gateway Protocol (BGP).

We study complexity and incentive-related issues in this model. Our main results are showing that in realistic and well-studied settings, BGP is incentive-compatible. I.e., not only does myopic behaviour of all players *converge* to a "stable" routing outcome, but no player has motivation to unilaterally deviate from the protocol. Moreover, we show that even *coalitions* of players of *any* size cannot improve their routing outcomes by collaborating. Unlike the vast majority of works in mechanism design, our results do not require any monetary transfers (to or by the agents).

Categories and Subject Descriptors

F.2.2 [**Analysis of Algorithms and Problem Complexity**]: Nonnumerical Algorithms and Problems
; C.2.2 [**Computer-Communication Networks**]: Network Protocols—*Routing Protocols*

General Terms

Economics, Theory

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BGP, Distributed Algorithmic Mechanism Design, Selfish Routing

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1. INTRODUCTION

The Internet is composed of smaller networks called *Autonomous Systems (ASes)*. ASes are owned by selfish, often competing, economic entities (Microsoft, AT&T, etc.). The task of establishing routes between ASes is called *interdomain routing*. Since not all ASes are directly connected, packets often have to traverse several ASes. The packets' routes are established via complex interactions between ASes that enable them to express preferences over routes, and are affected by the nature of the network (message delays, malfunctions, etc.). The standard interdomain routing protocol *de facto* is the *Border Gateway Protocol (BGP)*.

Routing Games. The first contribution of this paper is the presentation of a game-theoretic model of interdomain routing that captures many of its intricacies (e.g., the asynchronous nature of the network). In our model (as in [15, 14]), the network is defined by an undirected graph $G = (N, L)$. The set of nodes N represents the ASes, and consists of n *source-nodes* $1, ..., n$ (the players), and a unique *destination-node* d.[1] The set of edges L represents physical communication links between the nodes. Each source node i has a valuation function v_i that expresses a full-order of strict preferences over simple routes from i to d.

The model consists of two games: At the heart of the model is the sequential, asynchronous, and private-information Convergence Game, which is meant to model interdomain routing dynamics. Best-reply dynamics in the Convergence Game model crucial features of BGP dynamics, in which each AS is instructed to continuously execute the following actions:

- Receive update messages from neighbouring nodes announcing their routes to the destination.
- Choose a single neighbouring node whose route you prefer most (given v_i) to send traffic to.
- Announce your new route to all neighbouring nodes.

We also define a One-Round Game, which will function as an analytic tool. The One-Round Game can be regarded as the full-information non-sequential game underlying the Convergence Game. Pure Nash equilibria in the One-Round Game correspond to "stable solutions" in networking literature [15, 14], and are the "sinks" to which best-reply dynamics (BGP) can "converge".

[1]The reason for this is that in today's Internet, routes to each destination AS are computed independently.

We study several complexity and strategic problems in this model. Most importantly, we address the issue of incentive compatibility of best-reply dynamics in the CONVERGENCE GAME. We provide realistic settings in which the execution of best-reply dynamics (BGP) is in the best-interest of the players (ASes). We also address the following questions: How hard it is to establish whether a pure Nash exists in the ONE-ROUND GAME (the nonexistence of a pure Nash implies that best-reply dynamics will go on indefinitely)? How hard is it to get good approximations to the optimal social welfare?

Existence of Pure Nash Equilibria. Griffin and Wilfong have shown that determining whether a pure Nash equilibrium in the ONE-ROUND GAME (stable solution) exists is NP-hard [15]. Fabrikant and Papadimitriou [4] have strengthened their result to PSPACE-completeness.
We prove that this result extends to the communication model.

Theorem: *Determining whether a pure Nash equilibrium in the* ONE-ROUND GAME *exists requires exponential communication (in n) between the source-nodes.*

BGP Convergence and Incentives. Networking researchers, and others, invested a lot of effort into identifying sufficient conditions for the existence of a stable solution to which BGP *always* converges (see, e.g., [14, 24, 12, 11, 13, 5, 23, 4]). The most general condition known to guarantee this is "No Dispute Wheel", proposed by Griffin Shepherd and Wilfong [14]. No Dispute Wheel guarantees a *unique* pure Nash in the ONE-ROUND GAME, and convergence of best-reply dynamics to it in the CONVERGENCE GAME. No Dispute Wheel allows nodes to have significantly more expressive and realistic preferences than always preferring shorter routes to longer ones. In particular, a special case of No Dispute Wheel is the celebrated *Gao-Rexford setting* [12, 11] that is said to depict the commercial structure that underlies the Internet [19] (see Section 2 for an explanation about No Dispute Wheel and interesting special cases).

Feigenbaum, Papadimitriou, Sami, and Shenker [7] initiated an economic, or mechanism design, approach to interdomain routing. While BGP was designed to guarantee connectivity between trusted and obedient parties, in the age of commercial Internet these are no longer valid assumptions (ASes are owned by different economic entities with very different, and often conflicting, commercial interests). Identifying realistic settings in which BGP is incentive-compatible has become the paradigmatic problem in Distributed Algorithmic Mechanism Design (see [10] and references therein), and is the subject of many works [22, 6, 9, 8, 3, 10, 21, 16].

Recently, a step in this direction was taken in [8, 10]. It was shown that if No Dispute Wheel and an additional condition named *Policy Consistency* hold then BGP is incentive compatible in ex-post Nash. Informally, policy consistency means that *no* two neighbouring nodes disagree over which of *any* two routes is preferable – a severe restriction that does not necessarily hold in practice. We take a significant step forward by removing it (We still allow the Gao-Rexford commercial setting that does not imply Policy Consistency).

Unfortunately, we prove that best-reply dynamics are *not* incentive-compatible if Policy Consistency does not hold. This is true even if No Dispute Wheel holds, and can be shown to hold even in the Gao-Rexford setting.

Theorem: *Best-reply dynamics are not incentive-compatible in ex-post Nash even if the No Dispute Wheel condition holds.*

However, there is still hope for BGP. We consider a property called *"Route Verification"*. Route Verification means that a node can verify whether a route announced by a neighbouring node is indeed available to that neighbouring node (and if not simply ignore that route announcement). Unlike Policy Consistency, Route Verification *does not* restrict the preferences of ASes, but is achieved by modifying BGP (e.g., this can be achieved via cryptographic signatures). Achieving Route Verification in the Internet is an important agenda in security research[2]. Security researchers seek ways to implement Route Verification that are not only theoretically sound, but also reasonable to deploy in the Internet (see [2]).

We note that even if announcements of non-available routes are prevented by Route Verification, nodes still have many other forms of manipulation available to them: Pretending to have different preferences ("lying"), conveying inconsistent information (e.g., displaying inconsistent preferences over routes), denying routes from neighbours, and more. Hence, it still needs to be shown that Route Verification guarantees immunity of best-reply dynamics (BGP) to *all* forms of manipulation.

Our main result is the following:

Theorem: *Best-reply dynamics are incentive-compatible in (subgame-perfect) ex-post Nash if No Dispute Wheel and Route Verification hold.*

We stress that this result is achieved *without any monetary transfers* between nodes (as in [10], and unlike most prior works on interdomain routing, and in mechanism design in general).

Our result highlights an interesting connection between the two current research agendas that address the problem of disobedience and lack of trust in interdomain routing – security research and Distributed Algorithmic Mechanism Design. One of the implications of this result is that one can achieve incentive-compatibility in realistic settings (e.g., networks for which the Gao-Rexford constraints and Route Verification hold). This should further motivate security research, as it provides a strong strategic justification for modifications of BGP that guarantee Route Verification via cryptographic and other means (e.g., Secure BGP [2]).

In [10] the notion of *collusion-proofness in ex-post Nash* is defined. Informally, collusion-proofness in ex-post Nash means that a *group* of agents cannot collaborate to improve the outcome of any player in the group without strictly harming another player in the group. This means that the group as a whole has no interest to deviate from a strategy profile (at least one member will be harmed by doing so). The previous theorem can actually be strengthened to the following one:

Theorem: *Best-reply dynamics are collusion-proof in (subgame perfect) ex-post Nash if No Dispute Wheel and Route Verification hold.*

[2]"The US government cites BGP security as part of the national strategy for securing the Internet [Department of Homeland Security 2003]" [2]

In particular, this holds even for the coalition that contains all nodes. This implies that, if No Dispute Wheel holds, BGP is actually socially just in the following sense (also observed in [3]):

Corollary: *If No Dispute Wheel holds, the unique Nash equilibrium in the* One-Round Game *(to which best-reply dynamics always converge) is Pareto optimal.*

Maximizing Social Welfare. Finally, we turn our attention to the objective of maximizing the social-welfare, that has also been studied in the context of interdomain routing (see [9]). Maximizing the social welfare means finding a tree rooted in d, $T = R_1, ..., R_n$, in which node i is assigned route R_i, such that $\Sigma_i v_i(R_i)$ is maximized. In [8] it is shown that if No Dispute Wheel and Policy Consistency hold then BGP converges to a stable solution that also maximizes the social welfare. In contrast, we show that the removal of Policy Consistency can be disastrous in terms of welfare maximization. We do so by presenting two complementary bounds, one in the computational complexity model, and one in the communication complexity model.

Theorem: *Obtaining an approximation of $O(n^{\frac{1}{2}-\epsilon})$ to the optimal social welfare is impossible unless $P = NP$. Obtaining an approximation of $O(n^{1-\epsilon})$ to the social welfare is impossible unless $P = ZPP$. This holds for any $\epsilon > 0$ and even if No Dispute Wheel holds.*

Theorem: *Obtaining an approximation of $O(n^{1-\epsilon})$ to the optimal social welfare requires exponential communication (in n). This holds for any $\epsilon > 0$ and even if No Dispute Wheel holds.*

These two bounds actually hold even in the Gao-Rexford setting. These results should be compared with the previously known lower bound of $\Omega(n^{\frac{1}{2}-\epsilon})$ [9] (dependent on $P \neq ZPP$) for the case of general valuation functions. They show that even narrow conditions that ensure existence of pure Nash in the One-Round Game, and convergence of best-reply dynamics in the Convergence Game, might be very far from guaranteeing a good social welfare. A trivial matching upper bound of n exists even for general valuation functions (simply assign the node with the highest value for some route its most desired route).

1.1 Organization of the Paper

In Section 2 we present the model and the communication result about pure Nash equilibria. In Section 3 we present the results regarding incentives and best-reply dynamics in the Convergence Game. In Section 4 we present inapproximability results regarding social-welfare maximization. In Section 5 we discuss the open questions.

2. ROUTING GAMES

Here we define the game-theoretic model, and begin exploring the two games it contains.

2.1 Two Routing Games

In our model, the network is defined by an undirected graph $G = (N, L)$. N consists of n *source-nodes* $1, ..., n$ (the players), and a unique *destination-node* d. Each source node i has a valuation function v_i that assigns a non-negative value to every *possible* simple route from i to d (i.e., to every simple route in the complete graph over the nodes of G).[3] We make the standard assumption [14] that players have strict preferences: For any node i, and every two routes P, Q from i to d that do not have the same first link, it holds that $v_i(P) \neq v_i(Q)$. The model consists of two routing games:

2.1.1 A benchmark - the One-Round Game:

The One-Round Game is a game with full-information in which a strategy of a node i is a choice of an outgoing edge (i's choice of an AS to forward traffic to). The *payoff* of node i for a strategy profile is $v_i(R)$ if the strategies induce a route R from i to d, and 0 otherwise.

2.1.2 The Convergence Game:

The Convergence Game is a multi-round game with an infinite number of rounds. In each round one or more players (nodes) are chosen to participate by a *scheduler*. The scheduler models the asynchronous nature of the Internet, and decides which players participate in each round of the game. The *schedule* chosen must allow every player to play in an infinite number of rounds (the scheduler cannot deny a node from playing indefinitely). In each round of the game, a player i chosen to play can perform the following actions:

- Receive update messages from neighbouring nodes. Each update message announces a simple route from the sending neighbouring node to the destination.
- Choose a *single* outgoing edge $(i, j) \in L$ (representing a choice of a neighbouring node to forward traffic to), or $\emptyset$ (not to forward traffic at all).
- Announce simple routes (from i to d) to i's neighbouring nodes.

The scheduler decides in which round sent route announcements reach their destinations or if they will be dropped. It can arbitrarily delay update messages, but cannot indefinitely prevent update messages of a node from reaching its neighbour (see [14] for a formal model).

A *strategy* of a player in the Convergence Game specifies his actions in every round in which that player is chosen to participate. *Best-reply dynamics* is the strategy-profile in which every player continuously performs the following actions: Receive the most recent route announcements from *all* neighbours. Choose the neighbour with the *most* preferred simple route to d (according to your v_i). Announce this route to *all* neighbours.

If from some round onwards i's assigned route is constant then i's *payoff* is its value for that route. Otherwise, i's payoff is 0. More formally, the payoff of player i is $v_i(R)$ if R is a simple route from i to d and from some round onwards, for every link (r, s) on R, r *always* chooses s. Otherwise, i's payoff is 0. (All of our results hold even if when i's route "oscillates" i's payoff is set to be i's value for the highest valued route it is assigned an infinite number of times.).

[3]The reason for this is that nodes are not assumed to be familiar with the topology of the network and must be able to express preferences over *all* routes announced by other nodes.

2.2 Stable Solutions and Pure Nash Equilibria

Pure Nash equilibria in the ONE-ROUND GAME are known in networking literature as stable solutions. It is not hard to verify that each such stable solution forms a tree rooted in d. An important requirement from BGP is that it always converge to such a stable solution. However, this is not guaranteed in general, and definitely will not happen if a pure Nash does not exist. Griffin and Wilfong have shown that determining whether a pure Nash equilibrium in the ONE-ROUND GAME exists is NP-hard [15]. We strengthen this result by extending it to the communication model. The use of communication complexity for analyzing uncoupled Nash equilibrium procedures was recently presented in [17]. Our result can be seen as continuing this line of research.

THEOREM 2.1. *Determining whether a pure Nash equilibrium in the* ONE-ROUND GAME *exists requires exponential communication (in n).*

PROOF. We shall prove a reduction from the communication SET DISJOINTNESS problem (studied in [1]). In this problem, there are n communication parties. Each party i holds a subset A_i of $\{1, ..., K\}$. The goal is to distinguish between the two following extreme subcases:

- $\bigcap_i A_i \neq \emptyset$ (i.e., all parties have a mutual element)
- For every $i \neq j$ $A_i \cap A_j = \emptyset$ (i.e., no two parties share an element)

It is known [1] that in order to distinguish between these two subcases the parties must exchange $\Omega(K)$ bits (if $K >> n$). We set $K = 2^{\frac{n}{2}}$. The reduction to the problem of determining whether a pure Nash in the ONE-ROUND GAME exists is as follows: Consider a network with $2n$ source nodes and a unique destination node d. The set of nodes N consists of 2 disjoint subsets: n *sending nodes*, and n *transit nodes*. Each party $i \in [n]$ in the SET DISJOINTNESS problem is associated with a sending node s_i.

The transit nodes are divided into $\frac{n}{2}$ pairs $T_1, ..., T_{\frac{n}{2}}$. Each such pair of nodes T_r contains a specific node we shall call a 0-node and another node we shall call a 1-node. For every $r = 1, ..., \frac{n}{2} - 1$, each 0-node in T_r is connected to the 1-node in T_r, and to the 0-node in T_{r+1}. Similarly, each 1-node in T_r is connected to the 0-node in T_r, and to the 1-node in T_{r+1}. Both nodes in $T_{\frac{n}{2}}$ are connected to each other and directly to d. We divide the sending nodes into groups of size 3, $S_1, ..., S_{\frac{n}{3}}$. For every S_r, the 3 nodes in S_r are connected to each other, to the 0-node in T_1, and to d. See a description of the network in Figure 1. We shall refer to the 0-node in T_1 (which is the access point of all source-nodes to the transit nodes) as c.

We must now define the valuation functions of the different nodes. Let us start with the transit nodes: A 0-node in T_r u has a value of $\frac{1}{2}$ for any route to d in which the next-hop node after u is the 1-node in T_r, and a value of $\frac{1}{4}$ for routes in which the next-hop node after u is the 0-node in T_{r+1} (and very low values for all other routes to d, without ties). Similarly, a 1-node in T_r u has a value of $\frac{1}{2}$ for any route to d in which the next-hop node after u is the 0-node in T_r, and a value of $\frac{1}{4}$ for routes in which the next-hop node after u is the 1-node in T_{r+1} (and very low values for all other routes to d, without ties). Both nodes in $T_{\frac{n}{2}}$ prefer

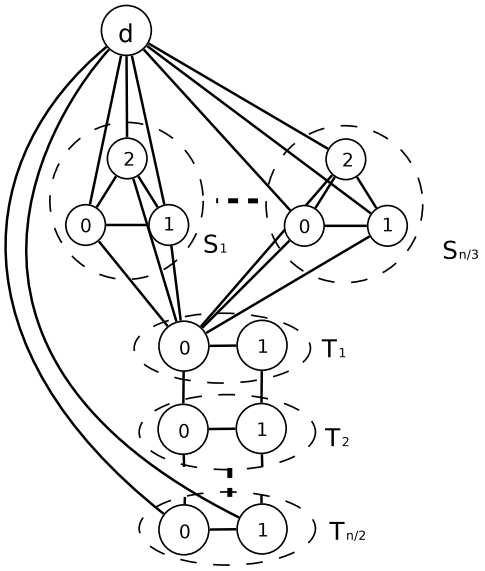


Figure 1: Network constructions used in the proof of Theorem 2.1

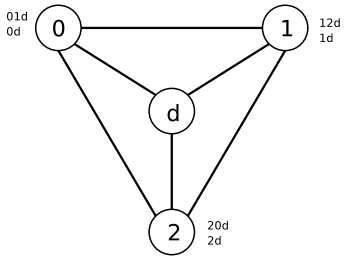


Figure 2: Bad Gadget

(a) Network structure for the proof (b) Bad Gadget

going through each other (a value of $\frac{1}{2}$) than directly to d (a value of $\frac{1}{4}$).

Fix a specific triplet of nodes S_r and let $0, 1, 2$ be those nodes. Each node $i = 0, 1, 2$ will assign a value of $\frac{1}{2}$ to the route $(i, i+1 \ (mod\ 3), d)$, a value of $\frac{1}{4}$ to the direct route to d, and very low values (without ties) to all other simple routes to d that only include nodes in S_r. This construction of S_r is known as BAD GADGET [15], and appears in Figure 2. BAD GADGET is an example of a small network in which no pure Nash equilibrium exists.

There are $2^{\frac{n}{2}}$ possible routes that go through the transit nodes and correspond to strings in $\{0,1\}^n$: Consider a string of bits $b = (b_1, ..., b_{\frac{n}{2}}) \in \{0,1\}^{\frac{n}{2}}$. b corresponds to the route in which the b_i-node in T_i forwards traffic to the other node in T_i. Fix an arbitrary order over these routes $R_1, ..., R_{2^{\frac{n}{2}}}$. Now, let each source node s_i assigns a value of 1 (no ties) to the route $((s_i, c)R_a$ iff $a \in A_i$. s_i assigns 0 to all unmentioned routes (observe that since all these routes go through the same next-hop node c breaking ties is not a problem).

The key point is that once the route of c is fixed, all source-nodes have no other choice of route through the tran-

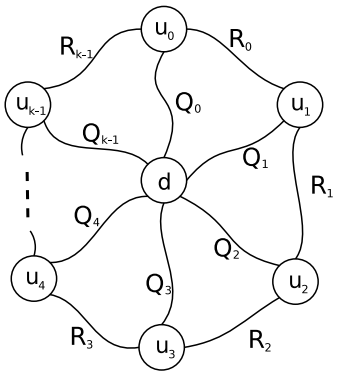
sit nodes. Hence, either all source nodes agree on a route that goes through the transit nodes, or some choose not to route through the transit nodes at all. The reader can verify that if there is some $a \in \bigcap_i A_i$ then assigning every node i the route $(s_i, c)R_a$ is a pure Nash equilibrium (and, in fact, a unique pure Nash equilibrium). If, on the other hand, for every $i \neq j$ $A_i \cap A_j = \emptyset$, then there is no pure Nash (because of the BAD GADGET construction for every triple S_r).

Hence, we have a network with $O(n)$ nodes, in which determining whether a pure Nash equilibrium exists is equivalent to solving the SET DISJOINTNESS problem with n players (each player i simulates node s_i), each holding a subset of $1, \ldots, 2^{\frac{n}{2}}$. It therefore requires at least $\Omega(2^{\frac{n}{2}})$ bits of communication. This concludes the proof of the theorem. □

REMARK 2.2. *The reader might be bothered by the fact that nodes are required to have preferences over very long routes (linear in n, that is, the number of all ASes in the entire Internet). We note that similar constructions in which the lengths of routes are asymptotically bigger than* $\log n$ *also lead to non-polynomial communication lower bounds.*

2.3 BGP Convergence and Best-Reply Dynamics

No Dispute Wheel is the broadest condition known, to date, to guarantee BGP convergence to a stable solution. In our terms, this translates to convergence of best-reply dynamics in the CONVERGENCE GAME. A Dispute Wheel, defined by Griffin et al. [14], is an abstract mathematical structure that can be induced by the network topology and the valuation functions. Formally, a dispute wheel is defined as the 3-tuple $(\mathcal{U}, \mathcal{R}, \mathcal{Q})$ where $\mathcal{U} = (u_0, u_1, \ldots, u_{k-1})$ is a sequence of k nodes in the network and $\mathcal{R} = (R_0, R_1, \ldots R_{k-1})$, $\mathcal{Q} = (Q_0, Q_1, \ldots, Q_{k-1})$ are sequences of routes that exist in G (indices for these nodes and routes should be considered modulo k). We shall call $u_0, ...u_{k-1}$ the *pivot-nodes*. It must hold that:



Each node u_i would rather route clockwise through node u_{i+1} than through the path Q_i

Figure 3: A Dispute Wheel

- Each route Q_i starts at u_i and ends at the destination node d.
- Each route R_i starts at node u_i and ends at node u_{i+1}.
- $v_i(Q_i) \leq v_i(R_i Q_{i+1})$ (where $R_i Q_{i+1}$ is the concatenation of the routes R_i and Q_{i+1})

The term "Dispute Wheel" is due to the resemblance of its form to that of a wheel. Figure 3 depicts such a structure. It is known that No Dispute Wheel guarantees that there is a unique stable solution [14].

No Dispute Wheel is known to hold for several interesting special cases. One special case is *Metric-Based Routing* (defined in [8]) which is a generalization of Shortest-Path Routing (in which the length of every link is 1).

In practice, there is no objective metric according to which all route choices are made. ASes' preferences are influenced by many economic, and other, considerations, like the business relationships between them. The Gao-Rexford setting [12] (see Appendix A) is said to accurately depict the underlying commercial structure of today's Internet [19], and is a special case of No Dispute Wheel [11]. In this setting, neighbouring ASes have one of two business relationships, based on long-term contracts: A *Customer-Provider* relationship (one AS purchases connectivity from another), or *Peering* (two ASes carry each other's transit traffic for free). These business relationships induce natural constraints (e.g., an AS cannot be an indirect customer of itself), formalized by Gao and Rexford.

3. BEST-REPLY DYNAMICS AND INCENTIVES

In this section we discuss several results regarding the incentive-compatibility of best-reply dynamics (BGP) in the CONVERGENCE GAME.

3.1 BGP Is Not Incentive-Compatible

We first prove the following theorem:

THEOREM 3.1. *Best-reply dynamics are not incentive-compatible in ex-post Nash even if the No Dispute Wheel condition holds.*

PROOF. Consider the network in Figure 4. There are 3 source nodes, $1, 2, m$ and a destination node d. Each nodes' two most preferred routes are listed next to it (where the higher route is more preferred than the lower route). It is easy to show that the valuation functions and topology of the network do not induce a Dispute Wheel (because route $2md$ does not actually exist). Hence, there is a unique pure Nash in the ONE-ROUND GAME, and best-reply dynamics always converge to this Nash in the CONVERGENCE GAME. The unique pure Nash is assigning $12d, 2d, m12d$ to $1, 2, m$, respectively. However, if m announces to 2 repeatedly that its route is md then it is easy to see that best-reply dynamics will always assign 1 the direct route to d, $1d$, thus enabling m to get its most preferred route $m1d$. Hence, deviating from best-reply dynamics (violating BGP) is beneficial.

We note that it is possible to define business relationships between the nodes in this example that are consistent with the Gao-Rexford constraints. □

3.2 BGP With Route Verification Is Incentive-Compatible

As we have said before, Incentive compatibility can be achieved if Route Verification holds.

THEOREM 3.2. *If No Dispute Wheel and Route Verification hold, then best-reply dynamics are incentive-compatible in (subgame perfect) ex-post Nash*

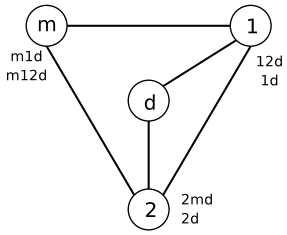


Figure 4: Best-Reply Dynamics Are Not Incentive-Compatible

PROOF. Consider a network graph $G = (N, L)$ for which No Dispute Wheel holds. There is a unique stable solution T to which all best-reply dynamics are bound to converge in the CONVERGENCE GAME. We denote the route of every source node r in T by T_r.

Assume, by contradiction, that some manipulating node r_m manages to reach a different outcome M by unilaterally deviating from best-reply dynamics (not executing BGP), and gains by doing so. We shall show that this implies the existence of a Dispute Wheel. The proof shall proceed in steps, pointing out a sequence of routes in the graph that will eventually form a Dispute Wheel.

We define the route M_r to be the route node r *believes* it is assigned in M. That is, it could be that the manipulator tricked nodes that send traffic through it in M to believe that their traffic is forwarded along a route not used in practice. We note that it could be the case that node r_m intentionally causes a protocol divergence that does not affect it, in order to improve its routing outcome (the choices of other nodes cause remote persistent route oscillations). If this is the case then the route M_r of a node r that is affected by this divergence, will simply be r's most preferred route, out of the routes assigned to r in the oscillation.

Since we assumed that r_m gained from its manipulation we deduce that:

$$v_{r_m}(T_{r_m}) < v_{r_m}(M_{r_m}) \tag{1}$$

Because r_m strictly prefers M_{r_m} to T_{r_m}, but did not choose it in the routing tree T, we must conclude that the route M_{r_m} is not available to r_m in T. This means that there must exist some node r (other than r_m) that is on the route M_{r_m} and that does not have the same route in M as it has in T. Let r_1 be the node on the path M_{r_m} that is closest to d on M_{r_m}, such that $M_{r_1} \neq T_{r_1}$.

By definition, all nodes that follow r_1 on the route M_{r_m} have exactly the same routes in T and in M. This means that the node r_1 could choose the route M_{r_1} in T. Since it did not choose that route we must conclude that:

$$v_{r_1}(M_{r_1}) < v_{r_1}(T_{r_1})^4 \tag{2}$$

We can now proceed to the next step in the proof. Since T_{r_1} is preferred by r_1, and was not chosen by r_1 in the routing tree M, it must be that it was not an available option. Therefore, there is some node r on the route T_{r_1}, that is not r_1, such that $T_r \neq M_r$. We select r_2 to be the node r closest to d on the path T_{r_1} for which $T_r \neq M_r$. As before, all nodes closer to d than r_2 on the route T_{r_1} send traffic along identical routes in both T and M. Hence, the route T_{r_2} must be available to r_2 even in M. The fact that it was not chosen in M implies that r_2 prefers M_{r_2} over it. Thus, we have that:

$$v_{r_2}(T_{r_2}) < v_{r_2}(M_{r_2}) \tag{3}$$

We can continue these steps, alternating between the routing trees T and M and creating a sequence of nodes as follows:

- $r_0 = r_m$

for $n = 0, 1, 2, \ldots$ we perform the following steps:

- **M step**: Let r_{2n+1} be the node r on the route $M_{r_{2n}}$ such that $M_r \neq T_r$, and r is closest among all such nodes to d on $M_{r_{2n}}$.
- **T step**: Let r_{2n+2} be the node r on the route $T_{r_{2n+1}}$ such that $M_r \neq T_r$, and r is closest among all such nodes to d on $T_{r_{2n+1}}$.

Note that the destination node d cannot appear in this sequence because the route $L_d = T_d$ is the empty set. Due to our construction, and to arguments similar to the ones presented before, the preferences over routes are as follows:

$$\text{for } i = 0, 2, 4, \ldots \qquad v_{r_i}(T_{r_i}) < v_{r_i}(M_{r_i}) \tag{4}$$

$$\text{for } i = 1, 3, 5, \ldots \qquad v_{r_i}(M_{r_i}) < v_{r_i}(T_{r_i}) \tag{5}$$

Since there is only a finite number of nodes, at some point a node will appear in this sequence for the second time. We denote the first node that appears two times in the sequence by u_0. Let $u_0, ..., u_{k-1}, u_0$ be the subsequence of $r_0, r_1, ...$ that begins in the first appearance of u_0 and ends in its second appearance. We shall examine two distinct cased.

CASE I: The manipulator r_m does not appear in the subsequence $u_0, ..., u_{k-1}, u_0$.

PROPOSITION 3.3. *If for all $i \in \{0, ..., k-1\}$ $r_m \neq u_i$ (the manipulator is not one of the nodes in the subsequence) then k must be even.*

PROOF. If k is odd, then it must be that u_{k-1} and u_0 (in its second appearance in the subsequence) were both selected in M steps, or were both selected in T steps. However, if this is the case we reach a contradiction as both nodes were supposed to be the node r closest to d on a certain route, such that $T_r \neq M_r$. Since $u_{k-1} \neq u_0$ this cannot be. □

If k is even then the subsequence of nodes $u_0, ..., u_{k-1}, u_0$, along with the T_{u_i} and M_{u_i} route, and the preferences over these routes (expressed before) form a dispute wheel (as in Figure 5).

CASE II: The manipulator r_m appears in the subsequence (that is, $u_0 = r_m$). We now need to handle two subcases: The subcase in which k is even and the subcase in which k is odd. If k is even then the second appearance of the manipulator (u_0) in the subsequence is due to a T step. If so, a Dispute Wheel is formed, as in the example in Figure 5[5].

[4]The reason that the inequality is strict is that equality can exist only if the two routes go through the same neighbouring node. This cannot be the case as $M_{r_1} \neq T_{r_1}$.

[5]The route $R \backslash S$ (where S is a sub-route of R) is route R truncated before the beginning of S

We are left with the subcase in which k is odd. In this case the second appearance of r_m was chosen in an M step. If so, it must be that $M_{u_{k-1}}$ (that goes through r_m) is not used in practice (otherwise, both u_{k-1} and that the second appearance of $r_m = u_0$ was chosen in M steps, and arguments similar to those of Proposition 3.3 would result in a contradiction). This must be the result of a manipulation by r_m. Let L_{r_m} be the false route reported by the manipulator to the node that comes before it on $M_{u_{k-1}}$. Recall that the manipulator can only announce a route L_{r_m} that exists and is available to it in M. Recall, that the second appearance of the manipulator was chosen due to an M step. Therefore, all nodes that follow it on $M_{u_{k-1}}$ (which are the same nodes as in L_{r_m} are assigned the same routes in T and M. Hence, L_{r_m} was available to r_m in T. It must be that $v_{r_m}(L_{r_m}) \leq v_{r_m}(T_{r_m})$, for otherwise r_m would have chosen L_{r_m} as its route in T (a contradiction to the stability of T). We know that $v_{r_m}(T_{r_m}) \leq v_{r_m}(M_{r_m})$ because we assumed that the manipulation performed by r_m was beneficial to it. We get:

$$v_{r_m}(L_{r_m}) \leq v_{r_m}(T_{r_m}) \leq v_{r_m}(M_{r_m}) \tag{6}$$

Thus, we form a Dispute Wheel with L_{r_m} as shown in Figure 6.

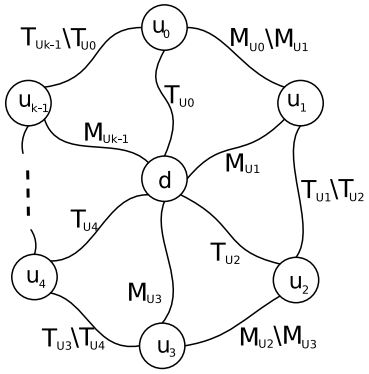


Figure 5: A Dispute Wheel constructed during the proof of Theorem 3.2 (even number of nodes).

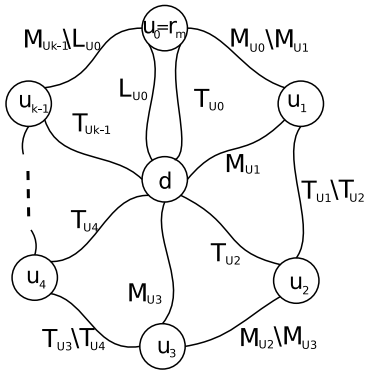


Figure 6: A Dispute Wheel constructed during the proof of Theorem 3.2 (odd number of nodes).

3.3 BGP With Route Verification Is Collusion-Proof

Theorem 3.2 can be strengthened as follows, using a similar construction:

THEOREM 3.4. *If No Dispute Wheel and Route Verification hold, then the best-reply dynamics are collusion-proof in (subgame perfect) ex-post Nash.*

PROOF SKETCH. We shall assume, by contradiction, that a group of manipulators colludes in an interdomain routing instance with no Dispute Wheel in order to improve their routing outcomes. We define T_r and M_r as in the proof of Theorem 3.2. We assume, by contradiction, that all manipulators are not harmed by this manipulation:

$$\forall v \in Manipulators \quad v_r(T_r) \leq v_r(M_r) \tag{7}$$

We shall arrive at a contradiction by showing the existence of a Dispute Wheel in a similar manner to that demonstrated in the proof of Theorem 3.2.

We begin the construction by selecting one of the manipulators that strictly gained from the collusion. We shall denote this manipulator by r_m (it must be that $T_{r_m} \neq M_{r_m}$). We then construct a sequence of nodes in a way similar to that explained in the proof of Theorem 3.2:

- $r_0 = r_m$
- **M step**: For a node r_n which is a manipulator we define r_{n+1} to be the node r on the route M_{r_n}, such that $M_r \neq T_r$, and r is the closest to d on M_{r_n} among all such nodes.
- **T step**: For a node r_n that is not a manipulator, and was chosen in an M step, we define r_{n+1} to be the node r on the route T_{r_n}, such that $M_r \neq T_r$, and r is the closest to d on T_{r_n} among all such nodes.
- **M step**: For a node r_n that is not a manipulator, and was chosen in a T step, we define r_{n+1} to be the node r on M_{r_n}, such that $M_r \neq T_r$, and r is the closest to d on M_{r_n} among all such nodes.

We define a subsequence of nodes $u_0, ...u_{k-1}, u_0$ as in the proof of Theorem 3.2. We now handle two cases. The first case is that no manipulator appears in $u_0, ..., u_{k-1}, u_0$. The handling of this case is precisely the same as in the proof of Theorem 3.2 (Case I).

The other case is that at least one of the nodes in $u_0, \ldots, u_{k-1}, u_0$ is a manipulator. First, we prove the following proposition:

PROPOSITION 3.5. *There is no $i \in \{0, \ldots, k-1\}$ such that both u_i and u_{i+1} (modulo k) are manipulators (no two manipulators come one after the other in the subsequence $u_0, ...u_{k-1}, u_0$).*

PROOF. By contradiction, let u_i and u_{i+1} be two consecutive manipulators. u_{i+1} was chosen in an M step. u_{i+1} is therefore the node r closest to d on M_{u_i} such that $M_r \neq T_r$. Hence, $M_{u_{i+1}}$ must be available to u_{i+1} in both M and T. We know that $v_{u_{i+1}}(T_{u_{i+1}}) \leq v_{u_{i+1}}(M_{u_{i+1}})$, as u_{i+1} is a manipulator. Since u_{i+1} chose $T_{u_{i+1}}$ over $M_{u_{i+1}}$ in T it must also be that $v_{u_{i+1}}(T_{u_{i+1}}) \geq v_{u_{i+1}}(M_{u_{i+1}})$. We conclude that $v_{u_{i+1}}(T_{u_{i+1}}) = v_{u_{i+1}}(M_{u_{i+1}})$. However, equality

of the values of routes assigned by u_{i+1} is only possible if u_{i+1} forwards traffic to the same node in both routes. Since both routes are available in T, this means that $T_{u_{i+1}} = M_{u_{i+1}}$. This contradicts the reason for which u_{i+1} was selected ($M_{u_{i+1}} \neq T_{u_{i+1}}$). □

The handling of the case in which at least one of the nodes in $u_0, ..., u_{k-1}, u_0$ is a manipulator, is very similar to CASE II in the proof of Theorem 3.2. The tricky part of the proof arises when a manipulator is selected in an M step. Due to Proposition 3.5, it must be that the node that precedes this appearance of the manipulator in the subsequence is not a manipulator. Such an event can be handled as the subcase in which k is odd in CASE II (in the proof of Theorem 3.2).

4. MAXIMIZING SOCIAL WELFARE

We prove that obtaining an approximation ratio better than n is hard even if No Dispute Wheel holds. In fact, this can be shown even for the Gao-Rexford setting. We present two lower bounds, one in the computational complexity model, and one in the communication complexity model.

THEOREM 4.1. *Obtaining an approximation of $O(n^{\frac{1}{2}-\epsilon})$ to the social welfare is impossible unless $P = NP$. Obtaining an approximation of $O(n^{1-\epsilon})$ to the social welfare is impossible unless $P = ZPP$. This holds for any $\epsilon > 0$ and even in the Gao-Rexford setting.*

PROOF. Our proof will be by reduction from CLIQUE. Assume a graph $G =< V, E >$, we construct a network with N nodes and L links. In this network, N consists of $2|V| + 1$ source-nodes and a unique destination node d. The source nodes are divided into 3 disjoint sets: Two sets N_1, N_2, such that $|N_1| = |N_2| = |V|$ and a *connection node* c. We associate a node $v(N_1) \in N_1$ and a node $v(N_2) \in N_2$ with every node $v \in V$. All nodes in N_1 are connected to the connection node c. All nodes in N_2 are connected to each other, to the connection node, and to d. See Figure 7.

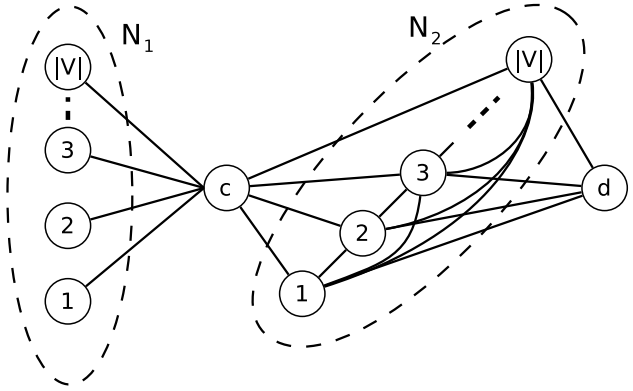


Figure 7: The network in the proof of Theorem 4.1

All nodes in N_2, and c, have valuation functions that assigns a value close to 0 to all routes (no ties). Fix some order O on the nodes in N_2. A node $v(N_1) \in N_1$ assigns a value close to 1 (no ties) to a route R iff *all* the following conditions hold:

- $(v(N_1), c)$ is the first link on R.
- The order of appearance of the nodes from N_2 in R is consistent with O.
- $v(N_2)$ is on R.
- For every node $u(N_2) \neq v(N_2) \in N_2$ on R there is an edge (v, u) in G.

Observe that since c is the only connection between N_1 and $N_2 \cup \{d\}$, c's route determines the routes of all nodes in N_1. The reader can verify that every clique in G corresponds to a routing tree with a social welfare that equals (almost) the size of the clique (assign every node in N_1 that is in the clique the route that goes through c and then the all the nodes in N_2 that are associated with nodes in the clique). In addition, the social welfare of every routing tree corresponds to a clique in the original graph G (the route from c to d through N_2 determines the identity of the clique). The theorem follows from the known inapprroximability results for CLIQUE [18].

We note that this result can be made to hold in the Gao-Rexford setting by defining business relationships as follows: c is a customer of all nodes in N_1. For every two nodes $r, s \in N_2$, such that s comes after r in O, s is r's customer. d is a customer of all nodes in N_2. □

THEOREM 4.2. *Obtaining an approximation of $O(n^{1-\epsilon})$ to the social welfare requires exponential communication (in n). This holds for any $\epsilon > 0$ and even in the Gao-Rexford setting.*

PROOF. The proof is similar to the proof of Theorem 2.1, and is by reduction from SET DISJOINTNESS. There are n parties, and each party i holds a subset A_i of $1, ..., K$. The goal is to distinguish between the two following extreme subcases:

- $\bigcap_i A_i \neq \emptyset$
- For every $i \neq j$ $A_i \cap A_j = \emptyset$

It is known that in order to distinguish between these two subcases the parties must exchange $\Omega(K)$ bits. We set $K = 2^{\frac{n}{2}}$.

Now, consider a network with $2n + 1$ source nodes and a unique destination node d. The set of nodes N consists of 3 disjoint subsets: *n sending nodes*, a *connecting node* c, and *n transit nodes*. Each party $i \in [n]$ in the SET DISJOINTNESS problem is associated with a sending node s_i.

The transit nodes are divided into $\frac{n}{2}$ pairs $T_1, ..., T_{\frac{n}{2}}$. All sending nodes are connected to the connecting node, which, in turn, is connected to both nodes in T_1. For every $r = 1, ..., \frac{n}{2} - 1$, each node in T_r is connected to both nodes in T_{r+1}. Both nodes in $T_{\frac{n}{2}}$ are connected directly to d. See a description of the network in Figure 8.

All transit nodes and the connecting node have a value close to 0 (no ties) for all routes. There are $2^{\frac{n}{2}}$ possible routes from c to d that go through the transit nodes (see proof of Theorem 2.1). Fix an arbitrary order on these routes $R_1, ..., R_{2^{\frac{n}{2}}}$. The valuation function of each s_i assigns a value close to 1 (no ties) to a route $(s_i, c)R_a$ iff $a \in A_i$. It assigns a value close to 0 to all other routes (no ties). The reader can verify that there is a route assignment with a social welfare-value close to n if there is some $a \in \bigcap_i A_i \neq \emptyset$ (assign every node i the route $(s_i, c)R_j$). If, on the other

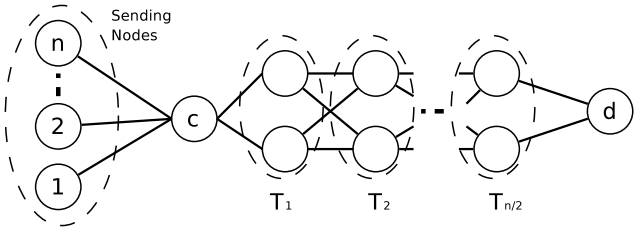


Figure 8: The network in the proof of Theorem 4.2

hand, for every $i \neq j$ $A_i \cap A_j = \emptyset$, then any route assignment cannot have a social-welfare value better than $1 + \epsilon$ (this is because c's route determines the routes of the sending nodes through the transit nodes).

Hence, we have a network with $O(n)$ nodes, in which determining whether the social-welfare is n or 1 is equivalent to solving the SET DISJOINTNESS problem with n players (each player i simulates node s_i), each holding a subset of $1, ..., 2^{\frac{n}{2}}$. It therefore requires at least $\Omega(2^{\frac{n}{2}})$ bits of communication. This concludes the proof of the theorem.

This result too can be made to hold for the Gao-Rexford setting if we define business relationships as follows: For every $i = 2, .., \frac{n}{2}$, the nodes in T_i are customers of the nodes in T_{i-1}. d is a customer of the nodes in $T_{\frac{n}{2}}$, the nodes in T_1 are customers of c, and c is a customer of all nodes in N_1. □

A trivial upper bound of n can be achieved by finding the node with the highest value for some route, assigning that route to that node (thus getting an n-approximation), and then assigning routes to all other nodes in a way that forms a tree rooted in d.

5. OPEN QUESTIONS

- No Dispute Wheel is sufficient, but not necessary, for guaranteeing BGP convergence. Find non-trivial characterizations of conditions that guarantee BGP convergence.
- What more general conditions than No Dispute Wheel and Route Verification guarantee incentive-compatible convergence of best-reply dynamics?
- Can we enforce No Dispute Wheel in an incentive-compatible manner? (We present a special case in which this is achieved via filtering in [20]).
- Identify realistic interdomain routing settings in which one can obtain a good approximation to the social-welfare.

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6. REFERENCES

[1] Noga Alon, Yossi Matias, and Mario Szegedy. The space complexity of approximating the frequency moments. *Journal of Computer and System Sciences*, 58(1):137–147, 1999.

[2] Kevin Butler, Toni Farley, Patrick Mcdaniel, and Jennifer Rexford. A survey of BGP security issues and solutions. Technical Report TD-5UGJ33, AT&T Labs, 2004.

[3] Ronny R. Dakdouk, Semih Salihoglu, Hao Wang, Haiyong Xie, and Yang Richard Yang. Interdomain routing as social choice. In *Proceedings the of Second International Workshop on Incentive-Based Computing*, Lisboa, Portugal, 2006.

[4] Alex Fabrikant and Christos Papadimitriou. The complexity of game dynamics: BGP oscillations, sink equilibria, and beyond. In *Proceedings of ACM-SIAM Symposium on Discrete Algorithms*, 2008.

[5] Nick Feamster, Ramesh Johari, and Hari Balakrishnan. Implications of autonomy for the expressiveness of policy routing. *SIGCOMM Computer Communication Review*, 35(4):25–36, 2005.

[6] Joan Feigenbaum, David R. Karger, Vahab S. Mirrokni, and Rahul Sami. Subjective-cost policy routing. *Theoretical Computer Science*, 378(2):175–189, 2007.

[7] Joan Feigenbaum, Christos H. Papadimitriou, Rahul Sami, and Scott Shenker. A BGP-based mechanism for lowest-cost routing. *Distributed Computing*, 18(1):61–72, 2005.

[8] Joan Feigenbaum, Vijay Ramachandran, and Michael Schapira. Incentive-compatible interdomain routing. In *Proceedings of the 7th ACM conference on Electronic commerce*, pages 130–139, 2006.

[9] Joan Feigenbaum, Rahul Sami, and Scott Shenker. Mechanism design for policy routing. *Distributed Computing*, 18(4):293–305, 2006.

[10] Joan Feigenbaum, Michael Schapira, and Scott Shenker. *Algorithmic Game Theory*, chapter 14, pages 363–383. Cambridge University Press, 2007.

[11] Lixin Gao, Timothy G. Griffin, and Jennifer Rexford. Inherently safe backup routing with BGP. In *Proceedings of the Twentieth Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM 2001)*, pages 547–556, 2001.

[12] Lixin Gao and Jennifer Rexford. Stable Internet routing without global coordination. *IEEE/ACM Transactions on Networking*, 9(6):681–692, 2001.

[13] Timothy G. Griffin, Aaron D. Jaggard, and Vijay Ramachandran. Design principles of policy languages for path vector protocols. In *Proceedings of the 2003 conference on Applications, technologies, architectures, and protocols for computer communications*, pages 61–72. ACM, 2003.

[14] Timothy G. Griffin, F. Bruce Shepherd, and Gordon Wilfong. Policy disputes in path-vector protocols. In *Proceedings of the Seventh Annual International Conference on Network Protocols*, pages 21–30. IEEE Computer Society, 1999.

[15] Timothy G. Griffin and Gordon Wilfong. An analysis of BGP convergence properties. *SIGCOMM Computer Communication Review*, 29(4):277–288, 1999.

[16] Alex Hall, Evdokia Nikolova, and Christos Papadimitriou. Incentive-compatible interdomain routing with linear utilities. In *Proceedings of the 3rd International Workshop on Internet and Network Economics, WINE 2007*, volume 4858 of *Lecture Notes in Computer Science*, pages 232–244. Springer, 2007.

[17] Sergiu Hart and Yishay Mansour. The communication complexity of uncoupled nash equilibrium procedures. In *Proceedings of the thirty-ninth annual ACM Symposium on Theory of Computing (STOC 2007)*, pages 345–353. ACM, 2007.

[18] Johan Hastad. Some optimal inapproximability results. *Journal of the ACM*, 48(4):798–859, 2001.

[19] Geoff Huston. Interconnection, peering, and settlements. In *Internet Global Summit (INET)*. The Internet Society, 1999.

[20] Hagay Levin, Michael Schapira, and Aviv Zohar. The strategic justification for BGP. Technical report, The Leibniz Center,The Hebrew University of Jerusalem, 2006.

[21] Ahuva Mu'alem and Michael Schapira. Setting lower bounds on truthfulness. In *Proceedings of the eighteenth annual ACM-SIAM symposium on Discrete algorithms (SODA 2007)*, pages 1143–1152. Society for Industrial and Applied Mathematics, 2007.

[22] Jeffrey Shneidman and David C. Parkes. Specification faithfulness in networks with rational nodes. In *Proceedings of the twenty-third annual ACM symposium on Principles of distributed computing (PODC 2004)*, pages 88–97. ACM, 2004.

[23] João L. Sobrinho. An algebraic theory of dynamic network routing. *IEEE/ACM Transactions on Networking*, 13(5):1160–1173, 2005.

[24] Kannan Varadhan, Ramesh Govindan, and Deborah Estrin. Persistent route oscillations in inter-domain routing. *Computer Networks*, 32(1):1–16, 2000.

APPENDIX

A. THE GAO-REXFORD FRAMEWORK

Studies of the commercial Internet [19] suggest two types of business relationships that characterize AS inter-connections: Pairs of neighbouring ASes have either a *customer-provider* or a *peering* relationship. Customer ASes pay their provider ASes for connectivity – access to Internet destinations through the provider's links and advertisement of customer destinations to the rest of the Internet. Peers are ASes that find it mutually advantageous to exchange traffic for free among their respective customers, *e.g.*, to shortcut routes through providers. An AS can be in many different relationships simultaneously: It can be a customer of one or more ASes, a provider to others, and a peer to yet other ASes. These agreements are assumed to be longer-term contracts that are formed because of various factors, e.g., the traffic pattern between two nodes.

In a seminal paper Gao and Rexford [12] suggest constraints on routing policies that are naturally induced by the business relationships between ASes.

No customer-provider cycles: Let G_{CP} be the directed graph with the same set of nodes as G and with a directed edge from every customer to its direct provider. We require that there be no directed cycles in this graph. This requirement has a natural economic justification as it means that no AS is indirectly its own provider.

Prefer customers to peers and peers to providers: *customer route* is a route in which the *next-hop* AS (the first AS to which packets are forwarded on that route) is a customer. *Provider* and *peer routes* are defined similarly. We require that nodes always prefer customer routes over peer routes, which are in turn preferred to provider routes. This constraint is on the valuation functions of the nodes – it demands every node assign customer routes higher values than peer routes, which should be valued higher than provider routes.

Provide transit services only to customers: Nodes do not always carry *transit traffic*—traffic that originates and terminates at hosts outside the node. ASes are obligated (by financial agreements) to carry transit traffic to and from their customers. However, ASes do not carry transit traffic between their providers and peers. Therefore, ASes should share only customer routes with their providers and peers but should share *all* of their routes with their customers. This constraint is on the filtering policy of the nodes – it requires that nodes only export peer and provider routes to their customers (customer routes are exported to all neighbouring nodes).